



Find the equation of the tangent line to the following curves:
Hint: Find the slope of the tangent line (IROC) first.

a) $y = x - x^3$ at $x = -1$

$$\begin{aligned} f(-1) &= -1 - (-1)^3 \\ &= -1 - (-1) \\ &= 0 \end{aligned} \quad (-1, 0)$$

$$\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$\lim_{x \rightarrow -1} \frac{x - x^3 - 0}{x + 1}$$

$$\lim_{x \rightarrow -1} \frac{x(1 - x^2)}{x + 1}$$

$$\lim_{x \rightarrow -1} \frac{x(1-x)(1+x)}{x+1}$$

$$\begin{aligned} m &= -1(1 - (-1)) \\ &= -1(2) \end{aligned}$$

$$m = -2$$

$$\begin{aligned} y &= mx + b \\ 0 &= -2(-1) + b \\ 0 &= 2 + b \\ b &= -2 \end{aligned}$$

$$y = -2x - 2$$

Calculus 120

Unit 1: Rate of Change and Derivatives

February 6, 2019: Day #5

1. Quiz

2. Assignments Due

Curriculum Outcomes

C1. Explore the concepts of average and instantaneous rate of change.

Determine the equation of the tangent line to the hyperbola

$y = \frac{1}{x}$ at the point $(-2, -1/2)$.

$$\lim_{x \rightarrow -2} \frac{f(x) - f(-2)}{x - (-2)}$$

$$\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \left(\frac{1}{2}\right)^x}{x+2}$$

$$\lim_{x \rightarrow -2} \frac{\frac{2}{2x} + \frac{x}{2x}}{x+2}$$

$$\lim_{x \rightarrow -2} \left(\frac{2+x}{2x} \right) \left(\frac{1}{x+2} \right)$$

$$\lim_{x \rightarrow -2} \frac{1}{2x}$$

$$m = -\frac{1}{4}$$

$$m = -\frac{1}{4} \quad P(-2, -\frac{1}{2})$$

$$y = mx + b$$

$$-\frac{1}{2} = \frac{-1}{4}(-2) + b$$

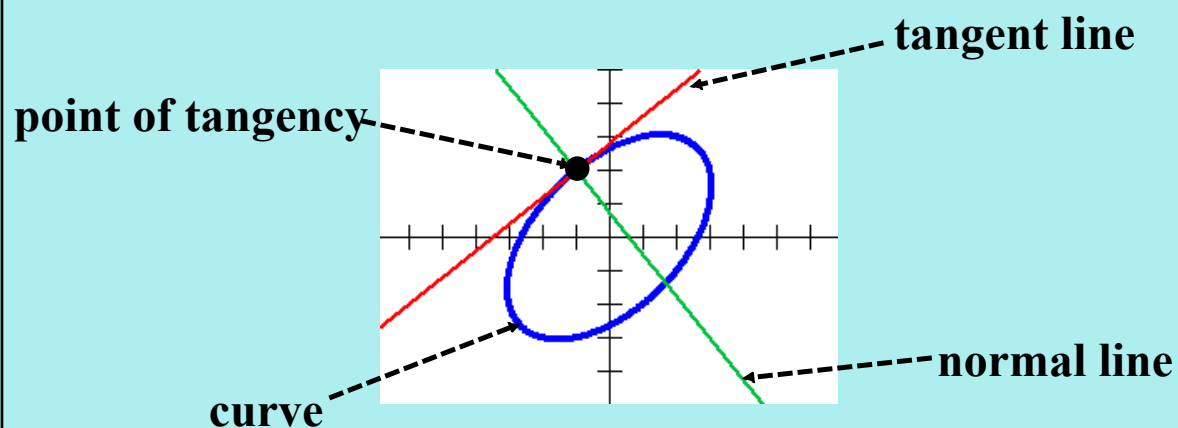
$$-\frac{1}{2} = \frac{1}{2} + b$$

$$-1 = b$$

$$y = -\frac{1}{4}x - 1$$

Normal Lines

The **normal line** to a curve at a point is the line perpendicular to the tangent at that point.



Remember from NRF....Perpendicular lines have slopes which are negative reciprocal (flip fraction, change sign)

Ex: For the function $f(x) = 4 - x^2$...

a) Determine the slope of the tangent line at $x = 1$.

b) Determine the equation of the tangent line at this point.

c) Determine the equation of the normal line at this point.

$$a) \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{4 - x^2 - 3}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{-x^2 + 1}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{-1(x^2 - 1)}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{-1(x-1)(x+1)}{x-1}$$

$$\begin{aligned} m &= -1(2) \\ m &= -2 \end{aligned}$$

$$\begin{aligned} f(1) &= 4 - 1^2 \\ f(1) &= 3 \end{aligned}$$

$$b) y = mx + b \quad \begin{aligned} m &= -2 \\ P(1, 3) \end{aligned}$$

$$3 = -2(1) + b$$

$$3 = -2 + b$$

$$b = 5$$

$$y = -2x + 5$$

$$c) m_{\perp} = \frac{1}{2} \quad P(1, 3)$$

$$y = mx + b$$

$$3 = \frac{1}{2}(1) + b$$

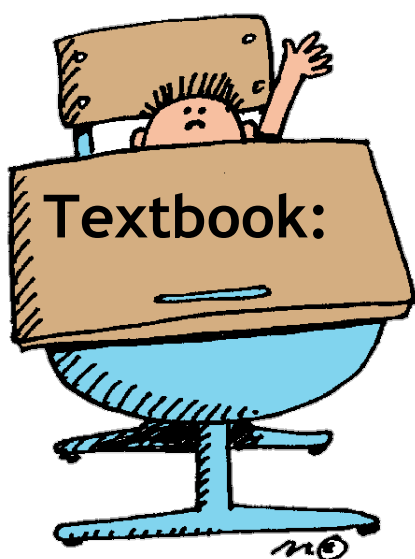
$$3 = \frac{1}{2} + b$$

$$\frac{5}{2} = b$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

Find the equation of the specified normal line to the following curve:

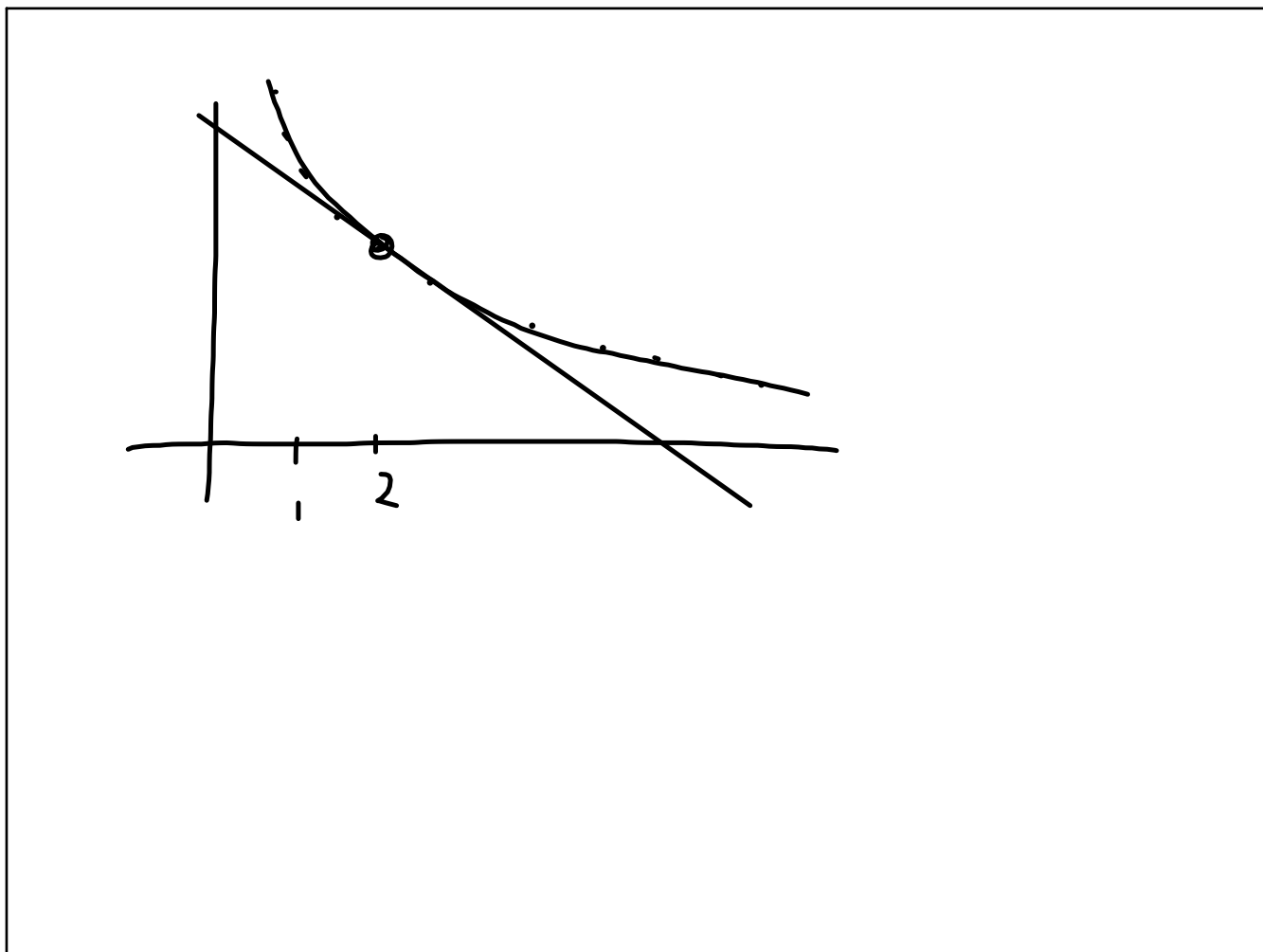
$$y = \frac{2}{x+1} \quad \text{at } x = -2$$

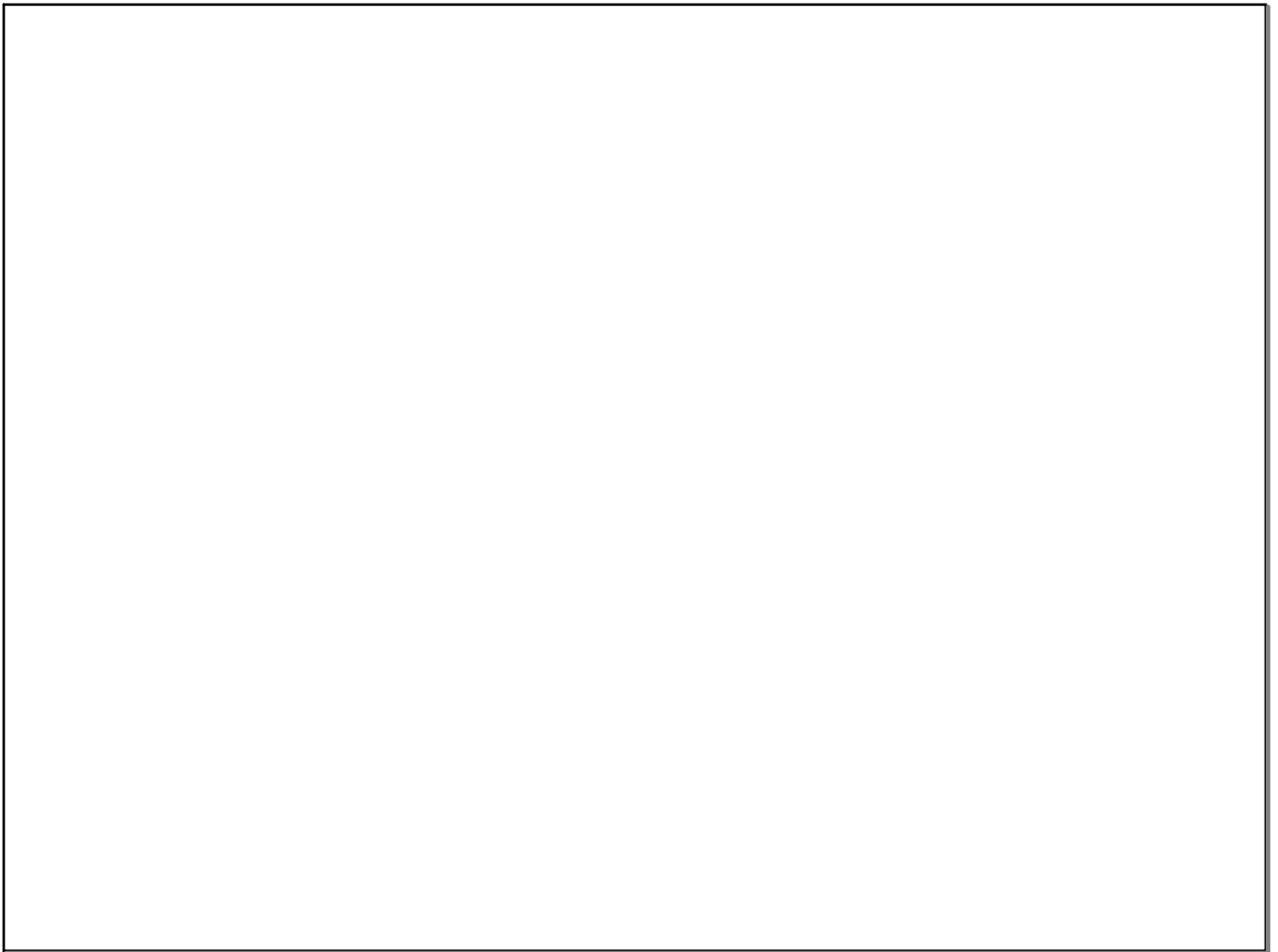


Practice



Page 92-93 #1a, 5a, 9, 10
11, 12, 23, 27, 29, 31





Attachments

2.1_74_AP.html



2.1_74_AP.swf



2.1_74_AP.html